

William Stanley Jevons and the Extent of Meaning in Logic and Economics¹

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Abstract

This paper shows that William Stanley Jevons was not precursor of logical positivism despite his attempt to build up a unified science. His mechanical reductionism was directed towards this project, and Jevons tried to found mathematics on logic through the development of a theory of number. We show that his attempts are unsuccessful, and that his errors remain visible within the totality of his mechanical system, including his economics. We argue that both his logic and his economics are comprehensible only when interpreted in terms of extent of meaning, and that Jevons' system gives rise to difficulties when interpreted in terms of intent of meaning. We argue that Jevons' methodological recommendations were intended to bridge the gap between extent and intent of meaning. Although Jevons did not succeed in establishing a unified science, his flawed methodology resulted in one of the first applications of statistics to the social sciences.

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1. Introduction

William Stanley Jevons (1835-1882) is a famous economist, statistician, logician, philosopher of science and even meteorologist. Although he is generally recognized as a forerunner of modern economics and statistics, his logic and philosophy of science are not mentioned very often. In this paper we argue that this may be explained by Jevons' somewhat contradictory position in the history of logic : he tries to found mathematics on logic, but his form of logic is inspired by the algebraic tradition of Boole and De Morgan. This contradiction results in Jevons' inability to establish a genuine definition of a 'unit'. Whereas Jevons is unable to found a truly Unified Science, he creates errors that remain visible within his mechanical system as a whole, especially in his economic theory.

In the second section we provide an overview of Jevons' reductionist world view. According to Jevons the laws of thought are concerned with detecting identity or similarity between certain objects of inquiry. A term is a name for a collection of objects, and it may be interpreted in two ways. Interpreted as regards intension, a term is a group of qualities; interpreted as regards extension, a term is a group of objects possessing those qualities. We argue that Jevons' system is comprehensible only when interpreted in terms of extent of meaning, and that it causes trouble when interpreted in terms of intent of meaning. Moreover, Jevons is unable to deal adequately with instances where interpretation in terms of intent of meaning is required (like in the case of the theory of number). Jevons defines 'partial identity' (or class inclusion) as a correlation of a group of objects with part of another group. Jevons states clearly that whole and part of a class are not identical, which implies that they may not be substitutes. Substantial and abstract terms are an exception, because these terms possess the quality of peculiar unity : parts and whole possess the same qualities in the first case, and it is impossible to draw a distinction between part and whole in the second. Jevons' intellectual programme is directed to the development of all possible instances of a general term, as is shown in the construction of his 'logical abacus'. Terms should be transformed until they refer to one or more concrete objects, which illustrates Jevons' preoccupation towards interpretation in terms of extent of meaning. This programme results in the project of a Unified Science, where logic forms the foundation for mathematics, and physics and economics are mathematical sciences. We demonstrate that this system fails because Jevons is unable to define the concept of a 'unit'.

In the third section we relate the problems in Jevons' logic to his economic thought. The 'extent of meaning' causes trouble for Jevons' economic theory when he discusses aggregates of individuals. These are treated as individuals as well, because both individuals and classes of individuals belong to the class of 'trading bodies'. Jevons' conception of the

'fictitious mean' forms the erroneous tool in order to bridge the gap between individual and aggregate, or between extent and intent of meaning.

The fourth section argues that Jevons' economic system should also be understood in terms of extent of meaning. Whereas in his logic all classes should be 'developed in extent' (transformed into a sum of concrete individuals that are only 'numerically different'), in economics all causal explanations should refer to individual motive forces. The interpretation in extent of meaning implies that Jevons has severe difficulties when discussing 'average' or 'aggregate' concepts. We argue that Jevons developed his statistical methodology in order to fill the gap between extent and intent of meaning : in the aggregate the 'disturbing causes' - the peculiarities of the individuals constituting the aggregate - balance.

We conclude that Jevons' position in the history of science is somewhat strange. He tried to establish a unified science, but his peculiar view on logic and mathematics prevented him to become a predecessor of logical positivism. Despite his contradictory conception of a 'unit', which remains visible within his economic theory, Jevons is still seen as a precursor of modern neoclassical economics. Moreover, Jevons developed some statistical tools in order to overcome the problems in his economic theory, and precisely these tools are the most important contribution of Jevons : he is one of the first economists to apply statistical techniques to data.

2. Jevons' Reductionist World View

Jevons' reductionist system is based on the laws of thought, which express the ability of mankind to discriminate, to detect identity, and to store these findings into memory. Robertson (1876:12) argues that Jevons passed very lightly over the question whether the laws of thought are subjective or objective.² Indeed, Jevons states "that logic treats ultimately of thoughts and things, and immediately of the signs which stand for them. Signs, thoughts, and exterior objects may be regarded as parallel and analogous series of phenomena, and to treat any one of the three series is equivalent to treating either of the other series" (Jevons 1873:9). The laws of thought are concerned with the products of mental reasoning, which implies that they belong to the branch of psychology. On the other hand, the laws of thought form the preconditions of reasoning, which indicates that they have to be impressed upon the mind. The laws of thought are therefore derived empirically from nature. Logic is an objective science, since it deals with laws which are both "in the

²George Croom Robertson (1842-92) was Professor of Logic and Mental Philosophy at University College, London, and editor of *Mind*. His criticism of Jevons' logic appeared in the first Volume of *Mind* (Black 1977:239).

nature of thought and things". Jevons refers to Leibniz' argument that we cannot prove the laws of thought, because they form the preconditions of all reasoning; and to Hartley's statement that we either have to accept the laws of thought, or that there would be no certainty whatsoever (Jevons 1873:8).

Jevons' position is normative : he argues that the laws of thought are part of an objectively existing Platonic world which cannot be fully grasped by the fallible and finite human mind (Jevons 1873:694). The human mind can make mistakes during the process of reasoning. We can, however, discover wrong assertions only when we can distinguish them clearly from all other assertions, which is only possible when the laws of thought are objectively true (Jevons 1873:6-9). Jevons seems to defend Descartes' (1637) view that everyone possesses the same faculty of reasoning, but not everyone makes always proper use of it. Since mistakes in reasoning are always possible, our thoughts cannot be the criterion of truth and the laws of thought govern the "events of objective nature".

The laws of thought deal with "identity, sameness, similarity, likeness, resemblance, analogy, equivalence or equality apparent between two objects" (Jevons 1873:1). A general analysis of human knowledge would then consist in pointing out the likeness of things (Black and Könekamp 1972:179). "Two objects are alike so far as when substituted one for another no alteration is produced, and *vice versa* when no alteration is produced by the substitution" (Jevons 1873:19). Jevons identifies three different laws of thought : the law of identity states that a thing is identical with itself; the law of non-contradiction holds that a thing cannot both be and not be; and the law of duality asserts that a thing either possesses a given attribute or does not possess it (Jevons 1869:111). These laws express the same truth. Jevons would like to reduce the laws of thought to one fundamental expression of truth, but he is unable to state the "characters of identity and difference in less than the threefold formula" (Jevons 1873:6). Symbolically, Jevons represents the laws as follows :

- (1) $A = A$
- (2) $Aa = 0$
- (3) $A = AB \cdot | \cdot Ab$

Jevons writes a for the negation of A , and $\cdot | \cdot$ denotes the inclusive disjunction. The central role of 'similarity' or 'equality' in Jevons' system is demonstrated by the use of the controversial symbol " $=$ ". Jevons argues that there is a strong analogy between logical propositions and mathematical equations, and therefore the same symbol should be applied in both cases (Jevons 1873:17). On the other hand, the "symbol $\cdot | \cdot$ is not identical with $+$, but is thus far analogous" (Jevons 1873:161). The central principle is called substitution of similars : the process of reasoning is nothing else than the replacement of a term in a

'logical equation' by its equivalent. The implication of this procedure is that the Aristotelian terms 'subject' and 'predicate' become indistinguishable, convertible and therefore useless (Jevons 1869:87). The symbols A and B refer to "terms" or "names", and "are symbols for the same object or group of objects"; they "represent a noun" and "stand for undetermined or unknown things" (Jevons 1873:13-6). Jevons' terms therefore represent two things at once : an abstract noun on the one hand, and an undetermined object or class of objects on the other hand. A class is a collection of objects, and a term is a name for a class. A name may denote some or more objects belonging to a class, or it may refer to certain common qualities possessed by those objects. "The objects denoted form the *extent* of meaning of the term; the qualities implied form the *intent* of meaning" (Jevons 1873:26). When adjectives are joined to a term its intent of meaning increases, but its extent of meaning decreases because it refers to less objects than before.

These modes of interpretation are essential to Jevons' criticism of Boole's system : "In his selective or class symbols, he [Boole] maintains the long standing confusion of quality and quantity thus x in his system means all things with the quality x , denoting the things in extension while connecting the quality in intension" (Grattan-Guinness 1991:24). Boole denotes by 1 all things of every quality; thus $x + (1-x)$ must make up 1. Multiplication (the logical 'and') of this expression of 1 with x leads to $x(x + 1 - x) = x + x - x$. Boole would cross out one $+x$ against $-x$, leaving x ; but Jevons argues that $x + x = x$ (the 'law of unity'), which implies that $x + x - x$ is really 0. According to Jevons Boole's method is true only when he uses " $1-x$ as the more qualitative contrary of x in intent of meaning only. The anomalies arise where $1-x$ is treated as the numerical complement of x in extent of meaning" (Grattan-Guinness 1991:24). We can extend this statement towards the claim that the introduction of Jevons' law of unity $A \cdot | \cdot A = A$ allows the interpretation of terms in extent of meaning. Indeed Jevons writes : "In extent $x + x$ means all x 's added to all x 's (...)" and if we "take all the x 's there can be no more left to add to them" (Grattan-Guinness 1991:25). Jevons seems to forget that his 'law of unity' causes trouble when interpreted in terms of intent of meaning. Boole, obviously bored with Jevons' letters, writes : "To be explicit, I now however reply, that it is not true that in Logic $x + x = x$ though it is true that $x + x = 0$ is equivalent to $x = 0$ " (Grattan-Guinness 1991:30). In his review of *The Principles of Science*, Robertson (1876:21) was also highly critical towards Jevons' use of $\cdot | \cdot$ and $=$: "Mr. Jevons is (...) anxious to extrude [the particular symbol $+$] from logic; but I do not see why it does not tell with equal force against the use of the symbol $=$, the true fount and origin of the evil against which he finds it thus necessary to protest".

Jevons is unable to deal with instances where interpretation in terms of intent of meaning is required. An example is Jevons' attempt to define 'number' through counting 'units' in space and time. When counting coins, every coin should receive a proper name :

we should count $C' + C'' + C''' + C'''' + \dots$. The coins are equal to each other (they all belong to the class C); they are different only because they reside on different points in space. Before counting, we should reduce all identical alternatives; but if the objects differ only quantitatively, we have to know the concept of quantity first before we can isolate this 'quality'. Jevons tries to reach the notion of 'number' through counting 'units' in space or time. "A unit is any object of thought which can be discriminated from every other object treated as a unit in the same problem" (Jevons 1873:157). The concept of 'unit' encounters some severe difficulties, as Frege notes :

Wenn wir mit 1 jeden der zu zählenden Gegenstände bezeichnen, so ist das ein Fehler, weil Verschiedenes dasselbe Zeichen erhält. Versehen wir die 1 mit unterscheidenden Strichen, so wird sie für die Arithmetik unbrauchbar.
 If we use 1 to stand for each of the objects to be numbered, we make the mistake of assigning the same symbol to different things. But if we provide the 1 with differentiating strokes, it becomes unusable for arithmetic. (Frege 1884:50, translated by J.L. Austin)³

As Schabas (1990:64) remarks, arithmetic is thereby derived in some mysterious way from the basic laws of logic. Jevons' defective approach gives rise to a symmetry between logic and mathematics, since he creates equations which have both a logical and a mathematical nature. Jevons defines (A) as the number of objects belonging to A, and if $A=B$ then it follows that $(A)=(B)$ (Jevons 1873:168-72). Jevons thus defines logic as an objective science resting upon the laws of thought, and bases mathematics (erroneously) on this logic. Jevons stresses the 'deep analogy' between mathematics and logic, but argues that logic forms the more fundamental science on which mathematics rests. We have to remark that Jevons (in contrast to Boole) wants to play down the use of algebra in logic. We will return to this issue in the last section.

The fact that Jevons wants to minimize the role of algebra in logic may explain why his theory of number, which is closely related to his interpretation in terms of extent of meaning, remains underdeveloped. Jevons acknowledges that whereas it is true in logic that $A \cdot | \cdot A = A$, it is absurd to say that in arithmetic $x + x = x$ (except when $x = 0$). According to Jevons we already "defined the units in one x as differing from those in the other" (Jevons 1873:162). This is comprehensible only when interpreted in extent of meaning : $2 + 2 = 4$ only because the first 2 and the second 2 denote different individuals; were they to denote the same individuals, $2 + 2$ would equal 2. Jevons does not see that

³Frege's solution to the problem is described clearly by Russell (1946:784). An instance of 'number' is a particular number like '3', and an instance of '3' is a particular triad. The triad is a plurality; the number '3' is a plurality of pluralities; and 'number' in general is a plurality of pluralities of pluralities. The concept of 'number' no longer is identified with 'plurality', as in Jevons.

when interpreted in intent of meaning, $2 + 2$ always equals 4, whether the first 2 and the second 2 refer to the same individuals or not.

The 'extent of meaning' is also visible in Jevons' discussion of wholes and parts. Jevons defines class inclusion or 'partial identity' as a correlation of one group of individuals with part of another group. The statement that all mammals (A) are vertebrates (B), is expressed as $A=AB$. The subject A denotes exactly the same individuals as AB : individuals which are mammals on the one hand, and individuals which are both vertebrates and mammals on the other. This is not an equality between classes as such (in modern symbols); the equality holds true because both terms can be reduced to the same individuals. "Cabinet Ministers are included almost always in the class Members of Parliament, because they are identical with some who sit in Parliament" (Jevons 1873:40-2, Mays and Henry 1953:169). The relation of whole and part is not one of identity : it is not permitted to transfer qualities from the whole to parts or vice versa. This does not apply to substantial and abstract terms : abstract terms, like 'redness', and substantial terms, like 'gold', possess the quality of absolute oneness. An abstract term is one and the same everywhere, and a substantial term refers to a substance that is one and the same everywhere. This implies that (in general) the qualities of a substance are present equally in parts and the whole, and in the case of abstract terms we cannot draw a distinction between parts and whole (Jevons 1873:28-30).

Jevons wants to represent all propositions in the forms of identities, including propositions concerning 'partial identity'. "*Same parts samely related make same wholes*. If, for instance, exactly similar bricks and other materials be used to build two houses, and they be similarly placed in each house, the two houses must be similar. There are millions of cells in a human body, but if each cell of one person were represented by an exactly similar cell similarly placed in another body, the two persons would be indistinguishable, and would be only *numerically* different" (Jevons 1873:19). Number is simply another name for diversity; exact identity is unity; and with difference arises plurality (Jevons 1873:157). Numerical (in)equality is therefore a special form of (in)equality, and quantity therefore a special quality. We can however define the special quality of 'quantity' only when we already possess the notion of quantity. Jevons therefore already presupposes the concept of number. We can relate this issue to Jevons' depiction of the fallible and finite human mind : man is 'logically weak' and has to count in time and space. Intellectual existences may be neither in space or time, but we have to represent the terms in categories of space and time. Space is not a necessary form of thought (Kant), but an impediment to logical reasoning (Jevons 1873:33-5, 769). A perfect intellect would therefore not need to count in space or time.

The 'extent of meaning' dominates Jevons' intellectual programme : "Having given any series of propositions we must be prepared to develop deductively the whole meaning embodied in them, and the whole of the consequences which flow from them" (Jevons 1873:12). He produces a table of permissible combinations of general terms, restricted to the conjunction : with n terms 2^n possible combinations can be constructed, since each term can either be affirmative or negative. Using this method, he develops a "logical abacus", which forms the starting point for later electronic analogues (Kneale and Kneale 1962:420-2). Starting from some qualities A, B, C, ... Jevons wants to 'fill up' logical space with all possibilities ABC, ABc, AbC, Abc, ... and then cross out all combinations which refer to a non-existing (group of) individual(s). Jevons' logic is therefore not only an objective, but also a mechanical science.

We conclude that Jevons' system is meaningful only when interpreted in terms of extent of meaning. A term denoting a quality should be reduced to a (group of) individual(s) possessing those qualities through 'deductive reasoning' in the form of the 'logical abacus'. Even the procedure of bifurcate classification should be formed on the principles of the logical alphabet. When dividing A into AB and Ab; AB into ABC and ABc; and ABc into ABcD and ABcd, it may appear that only ABC, ABcD, ABcd and Ab exist in reality. However, the combinations which are implied to be in reality are ABCd, ABcD, ABcd and Abcd. The bifurcate classification does not mention that ABCD, AbCD, AbCd and AbcD do not exist in nature (Jevons 1873:696). Jevons therefore carries out the development of 'the whole of the consequences' to the letter.

Jevons' goal is the development of a 'natural classification', based on definitions in which the meaning of all the words employed is made clear. A natural system is distinguished from an arbitrary or artificial system only in degree : the natural classification would reveal more 'resemblances' than the artificial one. The method of classification is related to the goal of the investigation, but Jevons seems to argue that there really exists *one* natural classification : "It is true that in the biological sciences there would be one arrangement of plants or animals which would be (...) in a certain sense natural, if it could be attained, (...) that *arrangement which would display the genealogical descent of every form from the original life germ*"(Jevons 1873:679-80).⁴ A 'summum genus' does not exist in Jevons' system, because it is impossible to think of an object or class without

⁴This fits into Mirowski's (1989:110-7) theory of stages : whereas anthropometric thinkers like Aristotle seek explanations which remain embedded in a specific context, the "hallmark of the lineamentric stage is the principle that systems of grouping and division are of overwhelming concern, while considerations of absolute magnitude are of secondary importance". In the syndetic stage "the ideal of unification is restored by means of a purely abstract, conventional standard". Jevons pleads for a natural classification in biology, which foreshadows "the stabilization of the metaphor of the gene" as this standard in biology.

simultaneously separating it from what is not that object or class. An 'infima species' is reached in the process of deductive reasoning when we arrive "at individual objects which are numerically distinct in the logical sense attributed to that expression in the chapter upon Number. Either, then, we must call the individual the *infima species* or allow that there is no such thing at all" (Jevons 1873:701-2). The fallible and finite human mind, restricted to counting in space and time, cannot hope to reach an encompassing 'natural classification'. What would Jevons have said about contemporary computers ? According to Mays and Henry (1953:167), "if Jevons were alive today it is unlikely that he would be surprised by modern digital computers and the arithmetical marvels which they perform". When discussing induction as the inverse operation of deduction, Jevons (1873:123) seems to hint at the late 20th century : "The work would probably occupy a good computer for many weeks, but it did not occupy me many minutes to multiply the factors together".

Jevons' quest for a natural classification is accompanied by an attempt to establish a Unified Science. Physics, as well as economics, are mathematical in nature, since they deal with quantities. This does not mean that they are exact sciences - there is no such thing, except in a comparative sense. The 'exactness' of a science depends on our ability to bring all relevant data more or less precisely into account (which is only approximately possible). Nobody doubts that the tides obey to some natural laws, but the complex contours of the seas do not allow numerical verification (Jevons 1871:3-7). As Schabas (1990:xii) states, Jevons was a "Pythagorean without the mysticism", in a "world ruled by number" portrayed as a huge mechanical system. Since logic is an objective and mechanical science, and mathematics is based on logic, and both economics and physics are mathematical sciences, all the branches mentioned are mechanical in nature. The 'substitution of similars' leads to a project of a Unified Science : "The whole value of science consists in the power which it confers upon us of applying to one object the knowledge acquired from like objects" (Jevons 1873:1). The detection of similarities gives rise to the discovery of an objectively existing Platonic world : "The constituents of the globe, indeed, appear in almost endless combinations; but each combination bears its fixed character, and when resolved is found to be the compound of definite substances" (Jevons 1873:2).

Jevons' ambitions are not restricted to the natural sciences. According to Jevons, economics deals with units of "pleasures" and "pains", which are difficult to conceive. But we can measure these units indirectly, since they prompt us to undertake actions like buying, selling, borrowing, lending, labouring, resting, producing and consuming. Even a physicist cannot directly measure gravitation, only the amount of motion it generates (Jevons 1871:11). Jevons' conception of 'mechanical psychological forces' is similar to the conception of gravitational force in physics - an analogy he borrowed from Richard Jennings (White 1994). Jevons' work contains a large amount of these analogies, which accounts for

his project of a Unified Science. Schabas (1990:84-9) argues that Jevons' theory of exchange, expressed in objective market mechanisms, relies heavily upon analogies with the theory of the lever. Jevons wants to reduce the observable to a more fundamental material world, an aim of science expressed in his description of economics as "the mechanics of utility and self-interest". Ideally, the prices are in direct proportion to the final degrees of utility, which can be derived from the fundamental mechanical forces. Unlike the classical approach, no mention is made of a 'natural' price around which the market price fluctuates : every price appearing in the market can be derived directly from some fundamental motive forces. White (1997) however demonstrates that Jevons did have a conception of 'natural prices', which express a long-term equilibrium in which all production factors are rewarded according to the amounts of utility they produce. This long-term equilibrium is equivalent to Jevons' 'natural classification' which cannot be reached by the finite human mind : the 'natural prices' would exist in the absence of mistaken decisions, monopolies, disturbing causes and institutions.⁵

In the next part of this paper we discuss Jevons' methodological recommendations, and we argue that they are an attempt to bridge the gap between extent and intent of meaning. The equality between terms as groups of qualities and terms as groups of objects possessing those qualities is reached through the establishment of the 'mean'.

3. Fixing the Fictitious Mean

Jevons (1873:359-60) identifies three different significations of the 'mean'. The 'fictitious mean' is simply a representative number serving as a convenient mode of comparing data with other series of quantities. The fictitious mean or average does not refer to a natural quantity. The 'precise mean' is approximately free from disturbing causes, and leads to a natural quantity; the 'probable mean' is more or less free from disturbing causes, and leads also approximately to a natural quantity. Jevons argues that the disturbing causes would balance in the second case, whereas the third use of the mean depends on the theory of probability. "Probability belongs wholly to the mind", or is "good sense reduced to calculation". Probability and chance do not exist in nature, but the fallible human mind is diluted with ignorance and has to quantify rational expectations by measuring the comparative amounts of knowledge and ignorance (Jevons 1873:197-200). Jevons' scientific programme is directed to the establishment of a 'natural classification', in which the constituents of the world bear their fixed characters. The fallible human mind

⁵See Peart (1995) for Jevons on errors and 'disturbing causes'.

can never grasp fully this natural classification, and therefore the theory of probability has an important place in science.

Whereas the precise and probable mean refer to natural quantities, the fictitious mean or average does not represent a really existing quantity. But the average is "of the highest scientific importance", because it enables us to simplify data. Jevons discusses the center of gravity within a body, which can be represented by the behaviour of one 'heavy' point; and the poles of a magnet, which do not refer to the ends of the magnet nor to any fixed points within (Jevons 1873:363-5).

The fictitious mean appears in Jevons' economic theory when he defines a 'trading body'. According to Jevons a trading body is any body of either buyers or sellers, which may consist of a single individual or of an aggregation of several individuals (the inhabitants of a specific country or continent, or the members of a certain trade). The mechanical principles of exchange remain true in every case : "We must use the expression with this wide meaning, because the principles of exchange are the same in nature, however wide or narrow may be the market considered. Every trading body is either an individual or an aggregate of individuals, and the law, in the case of the aggregate, must depend upon the fulfillment of law in the individuals" (Jevons 1871:89). A single individual does not vary his consumption by infinitesimal amounts according to each small change in price, but the aggregate will vary continuously. The laws of exchange are theoretically true in the case of the individual, and practically true in the case of the aggregate. Jevons remarks that the laws representing the conduct of an aggregate never represent exactly the conduct of a specific individual. Although this fictitious mean does not represent the character of an existing thing, average laws are useful "for the movements of trade and industry depend upon averages and aggregates, not upon the whims of individuals" (Jevons 1871:90).

As Schabas (1990:93-4) remarks, the treatment of the simple case of two trading bodies is justified by referring to the same practice in physics, where the 'three-bodies problem' has not been solved yet (Jevons 1873:760). Another justification by analogy concerns the use of the fictitious mean : like the center of gravity does not refer to a specific material point, the average member of a trading body does not refer to a specific individual. Jevons (1871:89-90) states clearly that the principles will always remain the same, but due to the impossibility of bringing all individual motive forces into account in the case of large trading bodies, the laws of economics can be held "practically true" by using the fictitious mean. The link with his general reductionist system is made explicit, since Jevons refers in the *Theory of Political Economy* to his discussion of the "fictitious mean" in *The Principles of Science* (1873:363).

We conclude that the laws of exchange remain true in the case of aggregates because they hold true for the individuals that are part of the aggregate trading body. This

explanation is related closely to Jevons' interpretation in terms of extent of meaning. A similarity between classes holds true only when they denote exactly the same individuals, and the exchange process between trading bodies follows the mechanics of utility and self-interest only when these mechanical laws are fulfilled in the several individuals constituting these trading bodies.

Jevons formalized the behaviour of two trading bodies. Suppose that the trade body A possesses the quantity a of corn, and that the trade body B possesses the quantity b of beef. After the exchange, A will hold $a-x$ of corn and y of beef, and B will hold x of corn and $b-y$ of beef. Jevons defines the final degree of utility as "meaning the degree of utility of the last addition, or the next possible addition of a very small, or infinitely small, quantity to the existing stock" (Jevons 1871:51). If the final degree of utility of corn is denoted by $\phi_1(a-x)$ for A and ϕ_2x for B, and the final degree of utility of beef is represented by ψ_1y and $\psi_2(b-y)$ for A and B respectively, then the quantities exchanged will satisfy two equations (Jevons 1871:98-100) :

$$\frac{\phi_1(a-x)}{\psi_1y} = \frac{y}{x} = \frac{\phi_2x}{\psi_2(b-y)}$$

These principles remain true whether the trading bodies under consideration consist of individuals or aggregates of individuals. In this last case, the 'aggregate' is the same as the 'average' (Jevons 1871:89), which implies that the functions ϕ_i and ψ_i are the utility functions of both the aggregate and the average. As the behaviour of the aggregate is equal to the behaviour of the average, the individual peculiarities disappear in both cases which implies that the 'disturbing' causes 'balance' (Peart 1995).

The Jevonian approach already came under criticism when Edgeworth (1881:29) discovered that the outcome of the exchange process would be indeterminate in the absence of bargaining, since all the points on the contract-curve would be possible solutions for the exchange equation.⁶ Peach (1987) raises the more important criticism that estimation of the amount of utility by 'reading back' from its 'quantitative effects' expressed in actions, cannot be carried out meaningfully without allowing inter-personal comparison of utility, a procedure forbidden by Jevons (Jevons 1871:14). But elsewhere Jevons explicitly compares the final degree of utility of money for poor and rich families (Jevons 1871:140-1); and concludes that it is difficult to inquire certain aggregate problems because of the "vast

⁶Edgeworth argues that Jevons' two equations are only one equation, namely

$$\frac{\phi_1(a-x)}{\psi_1y} = \frac{\phi_2x}{\psi_2(b-y)}$$

as the only condition which has to be fulfilled for the two variables x and y (Edgeworth 1881:20-1).

differences in the condition of persons" (Jevons 1871:148). We will not develop this already well-known criticism in further detail, but direct our attention to the connection with Jevons' general reductionist system.

By reducing the observable to a more fundamental world, Jevons assumes that every individual acts according to the same mechanical laws concerning utility and self-interest. The bargaining power is not taken into account in the discussion of trading bodies; the social positions of the different individuals disappear altogether; and, most fundamentally, *the different tastes and aspirations of individuals become equalized since they appear only as "fictitious mean" in the context of a large trading body.*

We can relate this issue to Jevons' failure to provide an adequate theory of number. Counting $C + C + C + C + \dots$ does not provide an acceptable foundation, since we cannot discriminate between the different C's. As in real life, where individuals are different, we should count $C' + C'' + C''' + C'''' + \dots$; but this is, as Frege notes, unusable in arithmetic theory. In physics, the mass points of a body are different only because they reside on different positions; the use of the "fictitious mean" does not bear important ontological implications. If we apply the same principle to economic theory, we are in fact equalizing different individuals and hence creating a poor theoretical 'fundamental' world. Later neoclassical movements can thus be seen as attempts to re-introduce real-life conditions in economic theory : e.g. the concept of bargaining power in Edgeworth or the theory of monopolistic competition by Chamberlin.⁷

Jevons (1871:93-5) states that his theory of exchange is concerned with static equilibrium analysis, but not with dynamics. Commodities are continually produced, exchanged and consumed; it should therefore be treated as a problem of dynamics. "But it would surely be absurd to attempt the more difficult question when the more easy one is yet so imperfectly within our power. It is only as a purely statical problem that I can venture to treat the action of exchange. Holders of commodities will be regarded not as continuously passing on these commodities in streams of trade, but as possessing certain fixed amounts which they exchange until they come to equilibrium". Once again, Jevons legitimates this approach by referring to current practice in physics (Schabas 1990:89-92). The analogy made here concerns the case of a moving pendulum : "It is much more easy to determine the point at which a pendulum will come to rest than to calculate the velocity at which it

⁷In the introduction of his path-breaking work, Chamberlin [1932:3] states : "Economic literature affords a curious mixture, confusion and separation, of the ideas of competition and monopoly. (...) Because *actual* competition (rarely free of monopoly elements) is supposedly explained by the theory of *pure* competition, familiar results really attributable to monopolistic forces are readily associated with a theory which denies them. This association of the theory of competition with facts which it does not fit has not only led to false conclusions about the facts; it has obscured the theory as well." (original emphasis) The same criticism applies to Jevons' account of the "trading bodies".

will move when displaced from that point of rest. Just so, it is a far more easy task to lay down the conditions under which trade is completed and interchange ceases, than to attempt to ascertain at what rate trade will go on when equilibrium is not attained" (Jevons 1871:94).

After having presented this analogy, Jevons goes on to develop his theory of exchange in which the simple case of two trading bodies is concerned. Due to the substitution of similars this approach holds true in the case of individuals as well as aggregates of individuals. But once again the analogy with the pendulum is deceptive : two mass points of the pendulum differ only in their position - changing the position of these points will not alter the position at which the pendulum will come to rest. In large trading bodies, consisting of different individual persons, the positions of these individuals certainly will affect the outcome of the exchange process : since every individual in the trading body has his/her own tastes and aspirations, the relative position might have its influence on the equilibrium.

Suppose two countries A en B involved in an exchange process. Country A is in possession of natural resources a, extraction of which would give rise to ecological or social problems. When the inhabitant of A leading the bargaining process prefers social and ecological stability to material values, the outcome might be that country B will have to offer more goods b in exchange for less goods a than otherwise. A relative change of the position of persons within the trading body hence will have its effects on the equilibrium position, unlike the change of position of two mass points in the pendulum. In other words : 'disturbing causes' do not balance in the case of large trading bodies.⁸ The statical approach hence includes the concept of "fictitious mean", which makes abstraction of the different persons involved in the trading body. The statical approach thus reinforces our statement that, through the substitution of similars, individuals as well as classes become equalized in economic theory as well. If Jevons' system really would confirm to real-life, it would not only imply inter-personal comparison of utility structures, but also the real

⁸Keynes (1930:71-8) criticized Jevons' procedure to 'average out' data round a 'mean' in the context of index numbers : "What is the flaw in the argument ? In the first place it is assumed that the fluctuations of individual prices round the 'mean' are 'random' in the sense required by the theory of the combination of independent observations. In this theory the divergence of one 'observation' from the true position is assumed to have no influence on the divergences of other 'observations'. But in the case of prices a movement in the price of one commodity necessarily influences the movement in the prices of other commodities, whilst the magnitudes of these compensatory movements depend on the magnitude of the change in expenditure on the first commodity as compared with the importance of the expenditure on the commodities secondarily affected. Thus, instead of 'independence', there is between the 'errors' in the successive 'observations' what some writers on probability have called 'connexity', or, as Lexis expressed it, there is 'sub-normal dispersion'." This criticism can be extended to Jevons' conception of a 'trading body'. See also Peart (1996:211).

existence of a "fictitious mean" incorporated in the person leading the bargaining process. The principle of marginal utility, derived from mechanical analogies of the pendulum or the lever, cannot be extended without problems to the case of trading bodies consisting of an aggregate of individuals, since the relative position of the individuals within the trading body has to be taken into account. The analogy with Jevons' unsuccessful 'theory of number' once again is striking. The gap between extent and intent of meaning cannot be filled by 'averaging out' 'disturbing causes'.

4. The Economics of the Extent of Meaning

The extent of meaning dominates not only Jevons' logic, but also his economic theory. Whereas in his logic all classes should be 'developed in extent' all causal explanations in economics should refer to individual motive forces. This applies also to the case of aggregate bodies, since these are (erroneously) identified with the individuals constituting the body because the 'disturbing causes' would balance anyway. Aggregates, or classes, are therefore depicted as individuals as well. We will now demonstrate that this individualistic attitude forms the starting point for Jevons' economic analysis; this attitude is absent in the writings of the classical economists Ricardo and Mill.

Ricardo (1821:69) defines rent as the compensation "which is paid to *the* owner of the land for the use of its original and indestructible powers" (my emphasis). He elaborates on the application of doses of capital to land; less capital "is the same thing as less labour" (Ricardo 1821:82). Ricardo refers to "labour" and "quantity of labour", but not to *the* labourer; on the other hand, he refers to *the* landlord and *the* cultivator (in fact the capitalist) (Ricardo 1821:74-5). Whereas the positions of landlord and capitalist are investigated from the individual's point of view, *labour* is treated as a homogeneous quality of which a certain quantity is applied to a particular piece of land. The same attitude is present in Mill (1848:429) : he refers to *the* farmer (or the capitalist), who receives profit, and to *the* landlord, who receives rent. On the other hand, Mill makes reference to "the labourers employed on" the land, and elaborates on what "the land can do for the labourers and for the capitalist" (Mill 1848:424, my emphasis). Both Ricardo and Mill therefore approach the position of landlord and capitalist from the individual's point of view, whereas 'labour' is treated as a homogeneous quality (equal to capital) and reference is made to the labourers (in plural).

Jevons (1871:216), on the contrary, supposes that "a certain labourer, or, what comes to exactly the same thing, a body of labourers, expend labour on several different pieces of land". Jevons therefore refers to individual labour, or to the labour of a body of labourers; these two are the same thing, since the 'body' is treated as an individual

(Mosselmans 1998). This 'individualistic attitude' coincides with Jevons' interpretation in terms of extent of meaning, and is absent in the writings of classical economists like Ricardo and Mill. We argued above that Jevons' 'individualism' is accompanied by illegitimate abstractions. We now relate some other problems for Jevons' economic theory to the interpretation in terms of extent of meaning. We discuss successively his inability to determine the average wage rate; his conception of an 'average period'; and his derivation of the "General Expression for the Rate of Interest".

Jevons is unable to determine the average wage rate. In *The Theory of Political Economy* Jevons refers always to *one* working man, and he is very reluctant to make statements concerning *the* rate of wages as a macro-economic phenomenon. From Jevons' individualistic point of view, it is of no use to make *theoretical* statements concerning the average wage rate, due to the large amount of different trades and skills. Instead, he thought that the average wage should be identified *statistically* (White 1991:158). However, Jevons' individualistic point of view implies that the collection of data is a very important problem : "While the theory is entirely mathematical in principle, I show at the same time how the data of calculation are so complicated as to be for the present hopeless" (Black 1973:410). Therefore, Jevons had great trouble with 'aggregate' or 'average' concepts; "he could not find the analytical means to link the depiction of wages as a residual in the capital analysis with his objective of explaining that income as the reflection of average labour productivity" (White 1991:157). In equilibrium and on the long run the labourer will be rewarded in accordance to his individual labour productivity. According to Jevons wages and profits can be seen as scarcity rents, similar to land rent (Mosselmans 1998). With Jevons' conception of a 'fictitious mean' or a 'body' of individuals in which 'disturbing causes balance' (Peart 1995), it is logically impossible to establish a functional relation between individual labour productivity and average wage rates. This requires statistical techniques and adequate data, which were in fact not available to Jevons (Aldrich 1987).

The same attitude can be seen in Jevons' description of the 'average period of production' or the 'average time of investment'. His definition is based on a simple agricultural example (1871:231), but Jevons is unclear about using the same definition in more complex cases. Elsewhere (1871:228-9), Jevons seems to use a "social average over all the commodities in question" (Steedman 1972:36-7). Again, Jevons is making illegitimate abstractions based on his belief of 'balancing disturbing causes' : he does not recognize that the concept of an 'average period' can be used only in a theory of interest without fixed capital and without compounded interest (Steedman 1972:50). The same can be said about Jevons' 'general expression for the rate of interest' : it is derived *as if* it would apply only to a single-commodity economy in a point-input, point-output production

process; but then he uses the formula *as if* it were not restricted to this special case. "His assumption was perhaps that what is true in a special case must also be true in the general case, provided that we reinterpret the terms of the special result as appropriate 'averages' of the corresponding terms of the general case" (Steedman 1972:40-4). Indeed : Jevons (erroneously) believes that the 'special case' is an adequate approximation, because it forms an 'average' or 'fictitious' mean in which the 'disturbing causes balance'.

The problems with 'aggregate bodies' are, at first sight, difficult to reconcile with the observation that Jevons was one of the first economists to deliberately apply statistical and probabilistic methods to economic problems. We should however keep in mind that most economic data available resisted the application of a standard technique; but also that these standard techniques often did not exist (Aldrich 1987). According to Peart (1993, 1995, 1996:173-93), Jevons' procedure of 'inductive quantification' entails a strong belief in the irrelevance of 'disturbing causes' (contrary to Mill). According to Jevons, the 'noxious' causes would 'balance', which means that they are not only quantitatively insignificant, but also distributed around a zero mean. Stigler (1982:362) regards the absence of a probabilistic analysis and measurement of the uncertainty remaining in the averages as an anomaly in Jevons' work. We conclude that Jevons' economics should be interpreted in terms of extent of meaning. Jevons' unsuccessful conception of 'balancing disturbing causes' functions as a methodological tool to bridge the gap between extent and intent of meaning.

In the last section of this paper we argue that Jevons' methodological recommendations and his mechanical logic, although filled with erroneous conceptions, foreshadows some important intellectual developments of the 20th century.

4. Logical Positivism, Statistics and Mathematical Economics

Grattan-Guinness (1988) identifies two traditions in the interactions between mathematics and logics : the algebraic tradition includes Boole and De Morgan; the mathematical tradition Peano and Russell. Jevons remained within the algebraic tradition, but tried to reduce the role of algebra, whereas Boole gave mathematics priority over logic, as it should be used to analyze the laws of thought (Grattan-Guinness 1988:75, 78). On the other hand, Russell tried to found mathematics on Peano's mathematical logic, but there is no connection between this tradition and Jevons' logic. Jevons tried to found mathematics on logic by using the concept of 'similarity' in his definition of a 'unit'. "Boole used mathematics to analyze (his form) of logic; at the other extreme, Russell claimed that only (his form of) logic was needed to analyze mathematics" (Grattan-Guinness 1988:78).

Jevons occupied a somewhat contradictory position in between of these opposites : he tried to found mathematics on logic, but his form of logic was inspired by the works of Boole and De Morgan. It was based on the principle of 'substitution of similars', and Jevons was unable to establish a genuine definition of a 'unit', as Frege's criticism shows. Moreover, similarity as such does not provide a satisfactory explanation, as Hempel and Oppenheim (1948:323) note : "The same point may be illustrated by reference to W. S. Jevons' view that every explanation consists in pointing out a resemblance between facts, and that in some cases this process may require no reference to laws at all and 'may involve nothing more than a single identity, as when we explain the appearance of shooting stars by showing that they are identical with portions of the comet'. But clearly, this identity does not provide an explanation of the phenomenon of shooting stars unless we presuppose the laws governing the development of heat and light as the effect of friction. The observation of similarities has explanatory value only if it involves at least tacit reference to general laws". Jevons did not bridge the gap between particular entities and the abstract notion of number, or between particular facts and a general law.

The methodological proposals of Paul Samuelson (1963) in economics are very similar to Jevons' ideas. According to Samuelson a theory should be identically equal to its empirical consequences. If A are the premises of a certain theory B, and C all the empirical consequences of B, then A, B and C express the same and are mutually interchangeable. If we start from some unrealistic premises A we can never reach plausible empirical consequences C. As Machlup (1964) remarks, Samuelson drops all theory since a theory is always broader than all its consequences taken together (Mosselmans 1997). As Jevons, Samuelson does not succeed in bridging the gap between particular entities and general laws. We conclude that Jevons had a logical positivist *attitude* (the project of a Unified Science, the reduction of the laws of thought to one fundamental expression, logic as the base of knowledge), but he used a logic from the algebraic tradition whereas a mathematical logic would be required.

Neurath (1983:67) praises Jevons' mechanical logic : "All this [physicalistical expression of equivalence] could be developed experimentally with the help of a 'thinking machine' as suggested by Jevons. Syntax would be expressed by means of the construction of the machine, and through its use, logical mistakes would be avoided automatically. The machine would not be able to write the sentence : 'two times red is hard'." On the other hand, Neurath (1970:1-27) criticizes Jevons for not applying his reasonings to all the sciences : "But neither Mill nor other thinkers of similar type [including Jevons] applied logical analysis consistently to the various sciences, thus attempting to make science a whole on 'logicalized' basis." We argued above that this last statement does not do justice to

Jevons' contribution to a 'logical' economic theory in which the extent of meaning is prevailing.

However, Jevons' main contributions to economics are his methodological proposals, which entailed the application of statistical techniques to the social sciences. Neurath (1987:133) writes : "Mill, Jevons, and Pearson were all very interested in the social sciences and tried to apply the empirical procedure to all questions without distinction. But none of these endeavours had the aim of making it possible for us to survey the wealth of our insight as a whole. Such a tendency is to be found in the prehistory of modern thought and is represented especially by the scholastics." Contemporary statistical techniques were unavailable for Jevons, and most economic data resisted the application of statistics. Moreover, Jevons' conception of the fallible and finite human mind puts restraints on the possibility to "survey the wealth of our insight as a whole". Although Jevons' statistics were erroneous in certain aspects, and did not succeed in bridging the gap between intent and extent of meaning, they formed the starting point for contemporary mathematical economics and econometrics (Morgan 1990:18-39). Unlike Mill, Jevons did not devote much attention to 'disturbing causes'. His belief that they would balance was erroneous, but it made the application of statistics to the social sciences possible : "As a statistician, Jevons played an important role in developing a conceptual approach to economic data that would later permit a quantification of uncertainty and the development of statistical laws for social science" (Stigler 1982:364).

6. Conclusion

We conclude that Jevons combined a logical positivist attitude with an encompassing tension between the extent and the intent of meaning. Both his logic and his economic theory are comprehensible only when interpreted in terms of extent of meaning. Trouble arises when interpretation in terms of intent of meaning is required. The 'fictitious mean' is an erroneous tool to resolve these problems, but it resulted in the application of statistical techniques to the social sciences. This leads us to a surprising conclusion : Jevons should not so much be honoured for the theories he developed, but for the errors he committed.

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